SOLVE $x' - 3x + 2y = \sin t$

$-4x + y' + y = -\cos t$

$\longrightarrow$ ① $(D-3)[x] + 2[y] = \sin t$

② $-4[x] + (D+1)[y] = -\cos t$

$(D+1)[①]$ $\quad (D+1)(D-3)[x] + 2(D+1)[y] = \cos t + \sin t$

$-2\,[②]$ $\quad 8[x] - 2(D+1)[y] = 2\cos t$

$+$

$(D+1)[\sin t] = (\sin t)' + \sin t$
$= \cos t + \sin t$

$((D+1)(D-3)+8)[x] = 3\cos t + \sin t$

$(r+1)(r-3)+8=0$

$r^2 - 2r + 5 = 0 \longrightarrow x'' - 2x' + 5x$

$(r^2-2r+1)+4=0$

$(r-1)^2 = -4$

$r-1 = \pm 2i$

$r = 1 \pm 2i$

$x_h = c_1 e^t \cos 2t + c_2 e^t \sin 2t$

$x_p = A\cos t + B\sin t$

$x_p' = B\cos t - A\sin t$

$x_p'' = -A\cos t - B\sin t$

$\cancel{x_p''} - 2x_p' + 5x_p \quad -2B\cos t + 2A\sin t + 5A\cos t + 5B\sin t = (4A-2B)\cos t + (2A+4B)\sin t$

$= 3\cos t + \sin t$

$4A - 2B = 3$

$2A + 4B = 1$

$*2$

$8A - 4B = 6$

$10A = 7$

$A = \frac{7}{10}$

$*(-2)$

$4A - 2B = 3$

$-4A - 8B = -2$

$-10B = 1$

$B = -\frac{1}{10}$

$x = \frac{7}{10}\cos t - \frac{1}{10}\sin t + c_1 e^t \cos 2t + c_2 e^t \sin 2t$

$4[①]$ $4(D-3)[x] + 8[y] = 4\sin t$

$(D-3)[②]$ $-4(D-3)[x] + (D-3)(D+1)[y] = +\sin t + 3\cos t$

$(D-3)[\cos t] = (\cos t)' - 3(\cos t)$
$= +\sin t + 3\cos t$

$((D-3)(D+1)+8)[y] = \cancel{3}5\sin t + 3\cos t$

$y_h = k_1 e^t \cos 2t + k_2 e^t \sin 2t$

$y_p = C\cos t + E\sin t$

$~~4C - 2E = +3~~$

$~~2C + 4E = \cancel{3}5~~$ $*2$ $*(-2)$

$~~8C - 4E = -6~~$

$~~10C = -3~~$

$~~C = -\frac{3}{10}~~$

$~~-4C - 8E = -6~~$

$~~4C - 2E = -3~~$

$~~-10E = -9~~$

$~~E = \frac{9}{10}~~$

$4C - 2E = 3$

$2C + 4E = 5$

$8C - 4E = 6$

$10C = 11$

$C = \frac{11}{10}$

$4C - 2E = 3$

$-4C - 8E = -10$

$-10E = -7$

$E = \frac{7}{10}$

$y = \cancel{-\frac{3}{10}}\frac{11}{10}\cos t + \cancel{\frac{9}{10}}\frac{7}{10}\sin t + k_1 e^t \cos 2t + k_2 e^t \sin 2t$

$x = \frac{7}{10}\cos t - \frac{1}{10}\sin t + c_1 e^t \cos 2t + c_2 e^t \sin 2t$

$$
\begin{array}{lcl}
-4x & & -\frac{28}{10}\cos t + \frac{4}{10}\sin t - 4c_1 e^t \cos 2t - 4c_2 e^t \sin 2t \\
+y' & = & \frac{\cancel{9}\;7}{10}\cos t + \frac{\cancel{3}\;-11}{10}\sin t + k_1 e^t \cos 2t - 2k_1 e^t \sin 2t \\
 & & \qquad + 2k_2 e^t \cos 2t + k_2 e^t \sin 2t \quad \Big] \; y' \\
+y & & \frac{\cancel{-3}\;11}{10}\cos t + \frac{\cancel{9}\;7}{10}\sin t + k_1 e^t \cos 2t + k_2 e^t \sin 2t \\
\hline
 & & -\cos t \quad + (-4c_1 + 2k_1 + 2k_2)\, e^t \cos 2t \quad = -\cos t \\
 & & \qquad + (-4c_2 - 2k_1 + 2k_2)\, e^t \sin 2t
\end{array}
$$

$$-4c_1 + 2k_1 + 2k_2 = 0 \longrightarrow c_1 = \frac{2k_1 + 2k_2}{4} = \frac{1}{2}(k_1 + k_2)$$

$$-4c_2 - 2k_1 + 2k_2 = 0 \longrightarrow c_2 = \frac{-2k_1 + 2k_2}{4} = \frac{1}{2}(k_2 - k_1)$$

$$\boxed{\begin{aligned} x &= \frac{7}{10}\cos t - \frac{1}{10}\sin t + \frac{1}{2}(k_1 + k_2)e^t \cos 2t + \frac{1}{2}(k_2 - k_1)e^t \sin 2t \\ y &= \frac{11}{10}\cos t + \frac{7}{10}\sin t + k_1 e^t \cos 2t + k_2 e^t \sin 2t \end{aligned}}$$

$$(D^2-D)[x] + D^2[y] = 2t \longrightarrow D(D-1)[x] + D^2[y] = 2t \quad (1)$$
$$(D^2-1)[x] + (D+1)[y] = -1 \longrightarrow (D+1)(D-1)[x] + (D+1)[y] = -1 \quad (2)$$

$(D+1)[(1)]$ $$D(D-1)(D+1)[x] + D^2(D+1)[y] = 2+2t$$

$-D[(2)]$ $$-D(D-1)(D+1)[x] - D(D+1)[y] = 0$$

$$(D+1)[2t] = (2t)' + 1(2t) = 2+2t$$
$$(-D)[-1] = -(-1)' = 0$$

$$(D^2-D)(D+1)[y] = 2+2t$$
$$(r^2-r)(r+1) = 0$$
$$r(r-1)(r+1) = 0 \longrightarrow r^3 - r = 0$$
$$y''' - y'$$
$$r = -1, 0, 1$$
$$y_h = c_1 + c_2e^t + c_3e^{-t}$$
$$y_p = (At+B)t$$
$$= At^2 + Bt$$
$$y_p' = 2At + B$$
$$y_p'' = 2A$$
$$y_p''' = 0$$

$$y_p''' - y_p' = -2At - B = 2+2t$$
$$-2A = 2 \quad -B = 2$$
$$A = -1 \quad B = -2$$

$$y = -t^2 - 2t + c_1 + c_2e^t + c_3e^{-t}$$
$$y' = -2t - 2 + c_2e^t - c_3e^{-t}$$
$$y'' = -2 + c_2e^t + c_3e^{-t}$$

$(D+1)[①] \quad D(D+1)(D-1)[x] + D^2(D+1)[y] = 2+2t$

$-D^2[②] \quad -D^3(D+1)(D-1)[x] - D^2(D+1)[y] = 0$

$(1-D)D(D+1)(D-1)[x] = 2+2t$

$D(D+1)(D-1)^2[x] = -2-2t$

$(r^2-r)(r^2-1)=0 \longleftarrow r(r+1)(r-1)^2=0$

$r^4-r^3-r^2+r=0 \qquad r=0,1,1,-1$

$x^{(4)}-x'''-x''+x'=0 \qquad x_h = k_1+k_2e^t+k_3te^t+k_4e^{-t}$

$x_p=(At+B)t$

$= At^2+Bt$

$x_p' = 2At+B$

$x_p'' = 2A$

$x_p^{(4)} = x_p''' = 0$

$x_p^{(4)} - x_p''' - x_p'' + x_p' = -2A + 2At+B$

$= 2At+(B-2A) = -2-2t$

$2A=-2 \qquad B-2A=-2$

$A=-1 \qquad B+2=-2$

$B=-4$

$x = -t^2-4t+k_1+k_2e^t+k_3te^t+k_4e^{-t}$

$x' = -2t-4+\underbrace{k_2e^t+k_3e^t}_{(k_2+k_3)e^t}+k_3te^t-k_4e^{-t}$

$x'' = -2+(k_2+2k_3)e^t+k_3te^t+k_4e^{-t}$

$$
\begin{aligned}
x'' &\quad \cancel{-2} + (k_2+2k_3)e^t + \cancel{k_3 te^t} + k_4 e^{-t} \\
-x' \quad = &\quad 2t + \cancel{4} + (-k_2-k_3)e^t - \cancel{k_3 te^t} + k_4 e^{-t} \quad = 2t \\
+y'' &\quad \cancel{+2} + c_2 e^t + c_3 e^{-t}
\end{aligned}
$$

$$2t + (c_2+k_3)e^t + (c_3+2k_4)e^{-t} = 2t$$

$$c_2+k_3=0 \qquad c_3+2k_4=0$$

$$c_2=-k_3 \qquad c_3=-2k_4$$

$$
\begin{aligned}
x'' &\quad -2 + (k_2+2k_3)e^t + \cancel{k_3 te^t} + k_4 e^{-t} \\
-x' &\quad = +\cancel{t^2} + 4\cancel{t} - k_1 + (-k_2 \quad)e^t - \cancel{k_3 te^t} - k_4 e^{-t} \quad = -1 \\
+y' &\quad -\cancel{2}t - 2 + (\quad c_2)e^t - c_3 e^{-t} \\
+y &\quad -\cancel{t^2} - \cancel{2}t + c_1 + c_{\cancel{1}2} e^t + c_3 e^{-t}
\end{aligned}
$$

← SUBSCRIPT ERROR (SUGGESTION: USE DIFFERENT LETTERS INSTEAD OF SUBSCRIPTS ON THE SAME LETTER)

$$(c_1-k_1-4) + \underbrace{(c_{\cancel{1}2}+c_2+2k_3)}e^t + \quad = -1$$

$$c_1-k_1-4=-1$$

$$c_1=k_1+3$$

$$c_{\cancel{1}2}+c_2+2k_3=0$$

$$\cancel{k_1+3-k_3+2k_3=0} \quad ?$$

$$2c_2+2k_3=0$$

$$c_2=-k_3$$

CONSISTENT

$$x = -t^2-4t+k_1+k_2e^t+k_3te^t+k_4e^{-t}$$

$$y = -t^2-2t+(k_1+3)-k_3e^t-2k_4e^{-t}$$